233. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Evaluate (*a*) $\int_0^{\frac{1}{2}\pi}\frac{\sin nx}{\sin x}dx$, and (b) $\int_0^{\frac{1}{2}\pi}\frac{\sin^2 nx}{\sin x}dx$, where n is a positive integer.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va., and J. SCHEFFER, A. M., Hagerstown, Md.

$$(a)\ \frac{\sin nx}{\sin x}=2[\cos x+\cos 3x+\cos 5x+\ldots+\cos(n-1)x],\ n \text{ even},$$

$$=1+2[\cos 2x+\cos 4x+\cos 6x+\ldots+\cos(n-1)x],\ n \text{ odd}.$$

$$\int_0^{\frac{1}{2}\pi}\frac{\sin nx}{\sin x}dx=2\left(\sin x+\tfrac{1}{3}\sin 3x+\tfrac{1}{5}\sin 5x+\ldots+\frac{1}{n-1}\sin(n-1)x\right)_0^{\frac{1}{2}\pi}$$

$$=2\left(1-\tfrac{1}{3}+\tfrac{1}{5}-\tfrac{1}{7}+\tfrac{1}{9}-\ldots\pm\frac{1}{n-1}\right),\ n \text{ even}=\tfrac{1}{2}\pi \text{ if } n=\infty.$$

$$\int_0^{\frac{1}{2}\pi}\frac{\sin nx}{\sin x}dx=\left(x+\sin 2x+\tfrac{1}{2}\sin 4x+\tfrac{1}{3}\sin 6x+\ldots\frac{2}{n-1}\sin(n-1)x\right)_0^{\frac{1}{2}\pi}$$

$$=\tfrac{1}{2}\pi \text{ when } n \text{ is odd}.$$

$$(b)\ \frac{\sin^2 nx}{\sin x}=\sin x+\sin 3x+\sin 5x+\ldots+\sin(2n-1)x.$$

$$\int_0^{\frac{1}{2}\pi}\frac{\sin^2 nx}{\sin x}dx=-\left(\cos x+\tfrac{1}{3}\cos 3x+\tfrac{1}{5}\cos 5x+\ldots+\frac{1}{2n-1}\cos(2n-1)x\right)_0^{\frac{1}{2}\pi}$$

$$=1+\tfrac{1}{3}+\tfrac{1}{5}+\tfrac{1}{7}+\ldots+\frac{1}{2n-1}.$$

Also solved by Prof. F. Anderegg.

DIOPHANTINE ANALYSIS.

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139. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

$2^{n-1}(2^n-1)$ is a multiply perfect number of multiplicity 2 when 2^n-1 is prime. Prove that there are no other multiply perfect numbers containing only 2 distinct primes.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $2^n-1=b$.

Then $\frac{2^n-1}{2^{n-1}(2-1)}\cdot\frac{b^2-1}{b(b-1)}=\frac{2^n-1}{2^{n-1}}\cdot\frac{b+1}{b}=\frac{2^n-1}{2^{n-1}}\cdot\frac{2^n}{2^n-1}=2.$

$\therefore 2^{n-1}(2^n-1)$ is a multiply perfect number of multiplicity 2. The second part of the problem is demonstrated in problem 137, Vol. XIII, Nos. 8-9.